Problem 1

Solve the cryptarithm

$$\begin{array}{ccccc} & T & I & G & A \\ + & T & I & G & A \\ \hline & E & N & A & M \end{array}$$

where different letters represent different digits, and no leading letter can be zero.

Solution

The cryptarithm is

$$\text{TIGA} + \text{TIGA} = \text{ENAM}.$$

Equivalently,

$$2 \cdot \text{TIGA} = \text{ENAM}.$$

Let c_1, c_2, c_3 be the carries from right to left. Since this is an addition of two numbers, each carry can only be 0 or 1:

$$c_1, c_2, c_3 \in \{0, 1\}.$$

Now write the equations column by column.

From the units column:

$$A + A = M + 10c_1.$$

So,

$$2A = M + 10c_1.$$

From the tens column:

$$G + G + c_1 = A + 10c_2.$$

So,

$$2G + c_1 = A + 10c_2.$$

From the hundreds column:

$$I + I + c_2 = N + 10c_3.$$

So,

$$2I + c_2 = N + 10c_3.$$

From the thousands column:

$$T + T + c_3 = E.$$

So,

$$2T + c_3 = E.$$

There is no extra digit in front of E, so the last column must not create another carry:

$$2T + c_3 < 10.$$

Now we choose digits step by step.

Start with the units column:

$$2A = M + 10c_1.$$

Choose

$$c_1 = 0.$$

Then

$$2A = M.$$

Choose

$$A = 2.$$

Then

$$M = 4.$$

So far,

$$A = 2, \qquad M = 4, \qquad c_1 = 0.$$

Now move to the tens column:

$$2G + c_1 = A + 10c_2.$$

Substitute $c_1 = 0$ and $A = 2$:

$$2G = 2 + 10c_2.$$

Choose

$$c_2 = 0.$$

Then

$$2G = 2.$$

So

$$G = 1.$$

So far,

$$A = 2, \qquad M = 4, \qquad G = 1.$$

Now move to the hundreds column:

$$2I + c_2 = N + 10c_3.$$

Substitute $c_2 = 0$:

$$2I = N + 10c_3.$$

Choose

$$c_3 = 1.$$

Then

$$2I = N + 10.$$

Choose

$$I = 5.$$

Then

$$2I = 10.$$

So

$$N = 0.$$

So far,

$$A = 2, \quad M = 4, \quad G = 1, \quad I = 5, \quad N = 0.$$

Now move to the thousands column:

$$2T + c_3 = E.$$

Substitute $c_3 = 1$:

$$2T + 1 = E.$$

Choose

$$T = 3.$$

Then

$$E = 2(3) + 1 = 7.$$

Therefore,

$$T = 3, \quad I = 5, \quad G = 1, \quad A = 2,$$
$$E = 7, \quad N = 0, \quad M = 4.$$

Now verify the full addition:

$$\text{TIGA} = 3512$$

and

$$\text{ENAM} = 7024.$$

Then

$$3512 + 3512 = 7024.$$

Therefore,

$$\boxed{\text{TIGA} + \text{TIGA} = \text{ENAM}}$$

is solved by

$$\boxed{T = 3,\ I = 5,\ G = 1,\ A = 2,\ E = 7,\ N = 0,\ M = 4.}$$

This matches the meaning of the words:

$$\text{tiga} + \text{tiga} = \text{enam},$$

which means

$$3 + 3 = 6.$$

But in the cryptarithm, the words become numbers:

$$3512 + 3512 = 7024.$$

Problem 2

Solve the cryptarithm

$$\begin{array}{rccccc} & & S & A & T & U \\ + & & T & I & G & A \\ \hline & E & M & P & A & T \end{array}$$

where different letters represent different digits, and no leading letter can be zero.

Solution

The cryptarithm is

$$\text{SATU} + \text{TIGA} = \text{EMPAT}.$$

This time the result has five letters, while both numbers being added have only four letters. Therefore, the first digit of the result must come from a carry.

So,

$$E = 1.$$

Let c_1, c_2, c_3, c_4 be the carries from right to left. Since the final result begins with E, we have

$$c_4 = E.$$

Therefore,

$$c_4 = 1.$$

Now write the equations column by column.

From the units column:

$$U + A = T + 10c_1.$$

From the tens column:

$$T + G + c_1 = A + 10c_2.$$

From the hundreds column:

$$A + I + c_2 = P + 10c_3.$$

From the thousands column:

$$S + T + c_3 = M + 10c_4.$$

Since $c_4 = 1$, the thousands-column equation becomes

$$S + T + c_3 = M + 10.$$

Now we choose digits step by step.

We already know

$$E = 1.$$

Look at the thousands column:

$$S + T + c_3 = M + 10.$$

This means $S + T + c_3$ must be at least 10. So S and T should be relatively large digits. Choose

$$c_3 = 0.$$

Then

$$S + T = M + 10.$$

Choose

$$S = 7, \qquad T = 6.$$

Then

$$S + T = 13.$$

So

$$M = 3.$$

So far,

$$E = 1, \quad S = 7, \quad T = 6, \quad M = 3, \quad c_3 = 0.$$

Now move to the units column:

$$U + A = T + 10c_1.$$

Substitute $T = 6$:

$$U + A = 6 + 10c_1.$$

Choose

$$c_1 = 0.$$

Then

$$U + A = 6.$$

We cannot use $1, 3, 6, 7$, because those digits are already used by E, M, T, S. Choose

$$A = 4, \qquad U = 2.$$

Then

$$U + A = 2 + 4 = 6.$$

So far,

$$E = 1, \quad S = 7, \quad T = 6, \quad M = 3, \quad A = 4, \quad U = 2.$$

Now move to the tens column:

$$T + G + c_1 = A + 10c_2.$$

Substitute $T = 6$, $c_1 = 0$, and $A = 4$:

$$6 + G = 4 + 10c_2.$$

If $c_2 = 0$, then

$$6 + G = 4,$$

which is impossible.

Therefore,

$$c_2 = 1.$$

Then

$$6 + G = 14.$$

So

$$G = 8.$$

So far,

$$E = 1, \quad S = 7, \quad T = 6, \quad M = 3, \quad A = 4, \quad U = 2, \quad G = 8.$$

Now move to the hundreds column:

$$A + I + c_2 = P + 10c_3.$$

Substitute $A = 4$, $c_2 = 1$, and $c_3 = 0$:

$$4 + I + 1 = P.$$

So

$$P = I + 5.$$

Choose

$$I = 0.$$

Then

$$P = 5.$$

Therefore,

$$S = 7, \quad A = 4, \quad T = 6, \quad U = 2,$$
$$I = 0, \quad G = 8, \quad E = 1, \quad M = 3, \quad P = 5.$$

Now verify the full addition:

$$\text{SATU} = 7462,$$

$$\text{TIGA} = 6084,$$

and

$$\text{EMPAT} = 13546.$$

Then

$$7462 + 6084 = 13546.$$

Therefore,

$$\boxed{\text{SATU} + \text{TIGA} = \text{EMPAT}}$$

is solved by

$$\boxed{S = 7,\ A = 4,\ T = 6,\ U = 2,\ I = 0,\ G = 8,\ E = 1,\ M = 3,\ P = 5.}$$

This matches the meaning of the words:

$$\text{satu} + \text{tiga} = \text{empat},$$

which means

$$1 + 3 = 4.$$

But in the cryptarithm, the words become numbers:

$$7462 + 6084 = 13546.$$

All Possible Solutions

All solutions for TIGA + TIGA = ENAM

There are 35 possible solutions.

For example,

$$3512 + 3512 = 7024$$

means

$$T = 3, \quad I = 5, \quad G = 1, \quad A = 2, \quad E = 7, \quad N = 0, \quad M = 4.$$

The complete list is:

1. $1349 + 1349 = 2698$
2. $1475 + 1475 = 2950$
3. $1524 + 1524 = 3048$
4. $1549 + 1549 = 3098$
5. $1649 + 1649 = 3298$
6. $1725 + 1725 = 3450$
7. $1762 + 1762 = 3524$
8. $1825 + 1825 = 3650$
9. $1862 + 1862 = 3724$
10. $1925 + 1925 = 3850$
11. $2175 + 2175 = 4350$
12. $2674 + 2674 = 5348$
13. $2687 + 2687 = 5374$
14. $2837 + 2837 = 5674$
15. $2937 + 2937 = 5874$
16. $3074 + 3074 = 6148$
17. $3087 + 3087 = 6174$
18. $3149 + 3149 = 6298$
19. $3274 + 3274 = 6548$
20. $3287 + 3287 = 6574$
21. $3425 + 3425 = 6850$
22. $3475 + 3475 = 6950$
23. $3512 + 3512 = 7024$
24. $3524 + 3524 = 7048$

25. $3549 + 3549 = 7098$
26. $3562 + 3562 = 7124$
27. $3649 + 3649 = 7298$
28. $3812 + 3812 = 7624$
29. $3825 + 3825 = 7650$
30. $3912 + 3912 = 7824$
31. $3925 + 3925 = 7850$
32. $4175 + 4175 = 8350$
33. $4325 + 4325 = 8650$
34. $4675 + 4675 = 9350$
35. $4825 + 4825 = 9650$

All solutions for SATU + TIGA = EMPAT

There are 20 possible solutions.

For example,

$$7462 + 6084 = 13546$$

means

$$S = 7, \quad A = 4, \quad T = 6, \quad U = 2, \quad I = 0, \quad G = 8, \quad E = 1, \quad M = 3, \quad P = 5.$$

The complete list is:

1. $3967 + 6529 = 10496$
2. $4583 + 8075 = 12658$
3. $4879 + 7508 = 12387$
4. $4956 + 5839 = 10795$
5. $5846 + 4938 = 10784$
6. $5879 + 7408 = 13287$
7. $5879 + 7608 = 13487$
8. $6835 + 3948 = 10783$
9. $6879 + 7508 = 14387$
10. $6934 + 3859 = 10793$
11. $7286 + 8042 = 15328$
12. $7462 + 6084 = 13546$
13. $7549 + 4805 = 12354$
14. $7659 + 5806 = 13465$
15. $7923 + 2569 = 10492$
16. $8549 + 4705 = 13254$
17. $8659 + 5706 = 14365$
18. $8934 + 3759 = 12693$
19. $8967 + 6429 = 15396$
20. $9286 + 8042 = 17328$

How to Choose Digits in a Cryptarithm

The main strategy is not to guess all digits randomly. Instead, we do this:

Choose a carry, check the column, then move to the next column.

For two-number addition, each carry is only 0 or 1. So every time we see a carry, we only need to test two possibilities:

$$c_i = 0$$

or

$$c_i = 1.$$

If a choice makes a digit repeat, gives a negative digit, gives a digit bigger than 9, or makes a leading letter equal to 0, then that choice is impossible. We go back and try another branch.

This method is called backtracking.

For example, in

$$\text{SATU} + \text{TIGA} = \text{EMPAT},$$

we found

$$E = 1$$

immediately because the result has five digits.

Then we chose

$$S = 7, \quad T = 6$$

because the thousands column needed a carry:

$$S + T + c_3 = M + 10.$$

After that, the remaining columns forced or strongly restricted the other digits.

So cryptarithm solving is not ordinary linear algebra. It is a combination of:

column equations

carry restrictions

digit restrictions

and

backtracking.

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Problem 3

Solve the cryptarithm

$$\begin{array}{ccccccc} & F & O & R & T & Y \\ & & & T & E & N \\ + & & & T & E & N \\ \hline & S & I & X & T & Y \end{array}$$

where different letters represent different digits, and no leading letter can be zero.

Solution

The cryptarithm is

$$\text{FORTY} + \text{TEN} + \text{TEN} = \text{SIXTY}.$$

This is interesting because the words also have the correct meaning:

$$40 + 10 + 10 = 60.$$

Let c_1, c_2, c_3, c_4 be the carries from right to left.
Since we are adding three numbers, each carry can be

$$c_i \in \{0, 1, 2\}.$$

Now write the column equations.
From the units column:

$$Y + N + N = Y + 10c_1.$$

Cancel Y from both sides:

$$2N = 10c_1.$$

From the tens column:

$$T + E + E + c_1 = T + 10c_2.$$

Cancel T from both sides:

$$2E + c_1 = 10c_2.$$

From the hundreds column:

$$R + T + T + c_2 = X + 10c_3.$$

So,

$$R + 2T + c_2 = X + 10c_3.$$

From the thousands column:

$$O + c_3 = I + 10c_4.$$

From the ten-thousands column:

$$F + c_4 = S.$$

Now solve step by step.

From the units column,

$$2N = 10c_1.$$

So either

$$N = 0, \quad c_1 = 0,$$

or

$$N = 5, \quad c_1 = 1.$$

Now check the second possibility. If $c_1 = 1$, then the tens column becomes

$$2E + 1 = 10c_2.$$

The left-hand side is odd, but the right-hand side is a multiple of 10. This is impossible.

Therefore,

$$N = 0, \quad c_1 = 0.$$

Now the tens column becomes

$$2E = 10c_2.$$

Since $E \neq N$, we cannot have $E = 0$. Therefore,

$$E = 5, \quad c_2 = 1.$$

Next, consider the ten-thousands column:

$$F + c_4 = S.$$

If $c_4 = 0$, then

$$F = S,$$

which is impossible because different letters must represent different digits.

Therefore,

$$c_4 = 1.$$

So,

$$S = F + 1.$$

Now use the thousands column:

$$O + c_3 = I + 10c_4.$$

Since $c_4 = 1$, this becomes

$$O + c_3 = I + 10.$$

The carry c_3 can only be $0, 1$, or 2.

If $c_3 = 0$, then

$$O = I + 10,$$

which is impossible.

If $c_3 = 1$, then we need

$$O + 1 \geq 10.$$

So $O = 9$, and then

$$I = 0.$$

But $N = 0$, so this repeats a digit. Therefore, this is impossible.

Thus,

$$c_3 = 2.$$

Then

$$O + 2 = I + 10.$$

So either

$$O = 8, \quad I = 0,$$

or

$$O = 9, \quad I = 1.$$

But $N = 0$, so $I = 0$ is impossible. Therefore,

$$O = 9, \quad I = 1.$$

Now we know

$$N = 0, \quad E = 5, \quad O = 9, \quad I = 1.$$

The remaining digits are

$$2, 3, 4, 6, 7, 8.$$

Now use the hundreds column:

$$R + 2T + c_2 = X + 10c_3.$$

Substitute $c_2 = 1$ and $c_3 = 2$:

$$R + 2T + 1 = X + 20.$$

So,

$$X = R + 2T - 19.$$

Also, from earlier,

$$S = F + 1.$$

So F and S must be consecutive digits.

From the remaining digits

$$2, 3, 4, 6, 7, 8,$$

the possible consecutive pairs for (F, S) are

$$(2, 3), \quad (3, 4), \quad (6, 7), \quad (7, 8).$$

Now test these possibilities in

$$X = R + 2T - 19.$$

(F,S)	Possible (T,R,X)
$(2,3)$	$(8,7,4)$
$(3,4)$	none
$(6,7)$	none
$(7,8)$	none

Therefore,

$$F=2, \quad S=3, \quad T=8, \quad R=7, \quad X=4.$$

Combining everything, we get

$$F=2, \quad O=9, \quad R=7, \quad T=8, \quad Y=6,$$

$$E=5, \quad N=0, \quad S=3, \quad I=1, \quad X=4.$$

Now verify:

$$\text{FORTY}=29786,$$

$$\text{TEN}=850,$$

and

$$\text{SIXTY}=31486.$$

Therefore,

$$29786+850+850=31486.$$

Hence,

$$\boxed{\text{FORTY}+\text{TEN}+\text{TEN}=\text{SIXTY}}$$

has the unique solution

$$\boxed{F=2,\ O=9,\ R=7,\ T=8,\ Y=6,\ E=5,\ N=0,\ S=3,\ I=1,\ X=4.}$$

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